Mathematics Syllabus

1. Linear algebra
2. Calculus
3. Matrices - system of linear eqn.
   - eigen values
   - eigen vectors
4. Differential calculus
5. Series
6. Partial differentiation
7. Application of partial differentiation

Matrices

Elementary transformation of matrices

These are performed on any non-zero matrix A.

1) R_{ij} - The interchange of i^{th} and j^{th} row.
   C_{ij} - The interchange of i^{th} and j^{th} column.

2) kR_i - The multiplication of each element of i^{th} row by a non-zero scalar quantity k
   kC_i - The multiplication of each element of i^{th} column by a non-zero scalar quantity k.

3) $(R_i + kR_j)$ - The multiplication of every element of j^{th} row by scalar k and adding element of i^{th} row.

   $(C_i + kC_j)$ - The multiplication of every element of j^{th} column by scalar k and adding element of i^{th} column.

Echelon form / cannonical form (changing complex to simple)

Let A be a matrix of order $m \times n$.
A is said to be in equivalent form when
1) The first row of A has the first ~~non-zero~~ element = 1.
   - it is called leading 1.

2) Below every leading 1, all elements are zero.

3) Leading 1 of every row lies to the right of the

leading ~~zero~~ 1 in the previous row.

4) All zero rows of A are in bottom

NOTE :- Every matrix can be reduced to equivalent form to solve system of linear eq^n

Rank of Matrix.

Number of non-zero rows in the equivalent form.

(total no. of rows) - (rows containing zeros) = (no. of non-zero rows)

Q. Find the rank ~~A~~ ~~wh~~ of matrix A where ρ.

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

Ans. $3 - 0 = 3$

Q. Find equivalent form of matrix A.

Ans. $R_2 - 3R_1 = R_2$

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & -10 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

$R_3 + R_1$

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & -10 \\ 0 & 2 & -3 & 10 \end{bmatrix}$$

Apply $R_3 + R_2 = R_3$

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

~~Apply~~ .

$3 - 1 = 2.$

$\rho(A) = 2$

Q. Find the rank of matrix A where

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

~~Apply $R_1 + R_2 \rightarrow R_2$~~

Apply R_{12}

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

Apply $R_3 - 3R_1 \rightarrow R_3$ and $R_4 - R_1 \rightarrow R_4$

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

Apply $R_3 - R_2 \rightarrow R_3$ and $R_4 - R_3 \rightarrow R_4$

$$A = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$4 - 2 = 2 = \rho(A)$

System of Linear eq^n

Consider a system of m linear eq^{n}s in 'n'-unknowns $x_1, x_2, x_3, \dots, x_n$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_n = b_3$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n = b_n$$

The above system of eq^n can be written in compact form by using matrix notation.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ & & \vdots & & \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

$$A \quad X = B$$

i.e. $AX = B$ where

Augmented Matrix

Adding one matrix in another.

$$[A/B] = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} & b_3 \\ & & \vdots & & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} & b_m \end{array}\right]$$

Non-homogeneous ~~matrices~~ eq^n

A system of eq^n ($AX = B$), if matrix B is not null matrix, then the system ($AX = B$) is called non-homogeneous.

Homogeneous eq^n

For the system of eq^n $AX = B$. when ~~matrix~~ B is a null matrix.

Example 1.

$$x + 2y + 3z = 7$$
$$x - 2y - 4z = 8.$$
$$2x + 3y - 9z = 2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & -4 \\ 2 & 3 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x & y & z \\ x & y & z \\ x & y & z \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 2 & -2 & 3 \\ 3 & -4 & -9 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 2 \end{bmatrix}$$

Ex. 2.

$$x - y + 3z = 0$$
$$8x + 7y - 9z = 0$$
$$3x - y + 6z = 0$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 8 & 7 & -9 \\ 3 & -1 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x & y & z \\ x & y & z \\ x & y & z \end{bmatrix} \begin{bmatrix} 1 & 8 & 3 \\ -1 & 7 & -1 \\ 3 & -9 & 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Finding sol^n of system of linear eq^n

1. Non-homogeneous eq^n.
   a) Write given ~~[illegible]~~ system in matrix notation $AX = B$
   b) Write augmented matrix. $[A/B]$

c) Reduce to echelon form.

d) Find $\rho(A)$ and $\rho(A/B)$.

e) Use the chart to decide the nature of the sol^n of the system.

$\boxed{AX = B}$

CONSISTENT
- unique sol^n
  - $\rho(A) = \rho(A/B) = \rho$
  - ρ = no. of var./unknowns.
- infinite sol^{n}s.
  - $\rho(A) = \rho(A/B) = \rho$
  - ρ < no. of variables/unknowns

INCONSISTENT
($\rho(A) \neq \rho(A/B)$)
$\downarrow$
no solution

Examine for consistency and solve if consistence

$$x + y + z = 3$$
$$2x - y + 3z = 1$$
$$4x + y + 5z = 2$$
$$3x - 2y + z = 4$$

To decide the consistency,

a) $$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 1 & 5 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}$$ as $AX = B$

b) $[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 1 & 3 & 1 \\ 4 & 1 & 5 & 2 \\ 3 & -2 & 1 & 4 \end{array}\right]$

c) Apply : $R_2 - 2R_1$, $R_3 - 4R_1$, $R_4 - 3R_1$

$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -5 \\ 0 & -3 & 1 & -10 \\ 0 & -5 & -2 & -5 \end{array}\right]$

~~Applly~~

Apply $R_3 + R_2$, $3R_4 - 5R_2$

$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -5 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & -11 & 10 \end{array}\right]$

Q Examine for consistency and solve ~~as~~ if consistent

$3x + y + 2z = 3$

$2x - 3y - z = -3$

$x + 2y + z = 4$

$AX = B$

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{array}\right]$$

R_{13}

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{array}\right]$$

$R_2 - 2R_1$ and $R_3 - 3R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & -5 & -1 & -9 \end{array}\right]$$

$-35 + 35$ $\quad$ -63 $+55$ 8

~~[A/B]~~ $7R_3 - 5R_2$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & 0 & 8 & -8 \end{array}\right]$$

$\rho(A/B) = 3 - 0 = 3$

$\rho(A) = 3 - 0 = 3$

$\therefore$ ~~Matrix~~ is Sol^n is consistent and unique.

$8z = -8$	$x + 2 - 4 - 1 = 4$
$z = -1$	$x + \cancel{3} = 4$
$-7y - 3z = -11$	$x = 9.$
$-7y + 3 = -11$	
$y = \cancel{14}\ 2.$	

Q. $x - y - z = 2$

$x + 2y + z = 2$

$4x - 7y - 5z = 2$

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \\ 4 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 1 & 2 & 1 & 2 \\ 4 & -7 & -5 & 2 \end{array}\right]$$

Apply $R_2 - R_1$ and $R_3 - 4R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 3 & 2 & 0 \\ 0 & -3 & -1 & -6 \end{array}\right]$$

Apply $R_3 + R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & -6 \end{array}\right]$$

$\rho(A) = 3 - 0 = 3$

$\rho(A/B) = 3 - 0 = 3 = \rho(A) =$ no. of variables

The matrix's solutions are consistent and unique

$z = -6$

$3y + (-12) = 0$

$y = 4$

$x - 4 + 6 = 2$

$x = 0$

Q. $2x - 3y + 5z = 1$

$3x + y - z = 2$

$x + 4y - 6z = 1$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 1 & -1 \\ 1 & 4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$A \quad X = B.$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & -3 & 5 & 1 \\ 3 & 1 & -1 & 2 \\ 1 & 4 & -6 & 1 \end{array}\right]$$

R_{13}

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 3 & 1 & -1 & 2 \\ 2 & -3 & 5 & 1 \end{array}\right]$$

$R_2 - 3R_1$ and $R_3 - 2R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & -11 & 17 & -1 \end{array}\right]$$

$R_2 - R_3$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2 = \rho(A/B) < 3$ (no. of variables)

Matrice's solution is consistent and infinite.

$-11y + 17z = -1$

$z = t$

$-11y + 17t = -1$

$17t = 11y - 1$

$t = \frac{11y-1}{17} \Rightarrow \frac{17t+1}{11} = y$

$x + 4y - 6t = 1$

$x + 4\left(\frac{17t+1}{11}\right) - 6t = 1$

$11x + 68t + 4 - 66t = 11$

$11x + 2t + 4 = 11$

$11x + 2t = 7$

$x = \frac{7-2t}{11}$

Q. Examine for consistency and solve if consistent.

a) $2x - y - z = 2$

$x + 2y + z = 2$

$4x - 7y - 5z = 2$

b) $5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$

Ans. a) $\begin{bmatrix} 2 & -1 & -1 \\ 1 & 2 & 1 \\ 4 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$A \quad X = B$

• $[A/B] = \left[\begin{array}{ccc|c} 2 & -1 & -1 & 2 \\ 1 & 2 & 1 & 2 \\ 4 & -7 & -5 & 2 \end{array}\right]$

Apply R_{12},

$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -1 & -1 & 2 \\ 4 & -7 & -5 & 2 \end{array}\right]$ X

Apply $R_2 - 2R_1$ and $R_3 - 4R_3$,

$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -5 & -5 & -2 \\ 0 & -9 & -9 & -6 \end{array}\right]$

30
-15

Apply ~~R_3~~ $5R_3 + 9R_2$

$[A/B] \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -5 & -5 & -2 \\ 0 & 0 & 0 & -12 \end{array}\right]$

$\rho[A/B] = 3 - 0 = 0$

$\rho(A) = 3 - 1 = 0$

$\rho(A) \neq \rho(A/B)$

∴ There are no solutions

b) $\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$

$A \quad X = B$

Date 9/9/23

$$[A/B] = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

~~Apply $C_1 - C_4$,~~

$$\cancel{[A/B] = \left[\begin{array}{ccc|c} 1 & 3 & 7 & 4 \\ -6 & 26 & 2 & 9 \\ -2 & 2 & 10 & 5 \end{array}\right]}$$

~~Apply $R_2 + 6R_1$ and $R_3 + 2R_1$~~

$$\cancel{[A/B] = \left[\begin{array}{ccc|c} 1 & 3 & 7 & 4 \\ 0 & 44 & 44 & 33 \\ 0 & 8 & 24 & 13 \end{array}\right]}$$

Apply $2R_1 - 3R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -72 & 8 & 19 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

Apply $R_2 - 3R_1$ and $R_3 - 7R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -72 & 8 & 19 \\ 0 & 242 & -22 & -48 \\ 0 & 506 & -46 & -128 \end{array}\right]$$

Apply $506R_2 - 242R_3$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -72 & 8 & 19 \\ 0 & 242 & -22 & -48 \\ 0 & 0 & 0 & 16688 \end{array}\right]$$

$\rho(A/B) = 3 - 0 = 3$

$\rho(A) = 3 - 1 = 2$.

$\rho(A) \neq \rho(A/B)$

$\therefore$ There is no sol^n

Non-homogeneous eq^n involving variables

Q. Investigate the values λ and μ so that eq^n s

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$ ... has :

i) no sol^n ii) Unique sol^n

iii) infinite nos of sol^n

Ans. STEP 1: The given system can be written as $AX = B$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$

STEP 2: Consider $[A/B]$.

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 3 & \lambda & \mu \end{array}\right]$$

STEP 3: Convert $[A/B]$ in equivalent form.

Apply $R_2 - R_1$ and $R_3 - R_1$;

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & -\lambda-1 & \mu-6 \end{array}\right]$$

Apply $R_3 - R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right] \text{ ——— (1)}$$

From (1), there is no sol^n if $\rho(A) \neq \rho(B)$

$\therefore$ $\lambda = 3$ & $\mu \neq 10$ OK

~~$\lambda \neq 3$ & $\mu \neq 10$~~

From ①, there can be a unique solⁿ if

$\rho(A) = \rho(A/B)$ = no. of variables

$\lambda \neq 3$ and $\mu \neq 10$.

From ③, there can be infinite solutions if

$\rho(A) = \rho(A/B) <$ no. of variables.

$\lambda = 3$ and $\mu = 10$.

Q. Investigate the value of λ and μ so that

$2x - y + 3z = 2$

$x + y + 2z = 2$

$5x - y + \lambda z = \mu$ ... has

a) no solⁿ

b) unique solⁿ

c) infinite solⁿ.

Ans.
$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 2 \\ 5 & -1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ \mu \end{bmatrix}$$

$$A \quad X = B$$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 1 & 1 & 2 & 2 \\ 5 & -1 & \lambda & \mu \end{array}\right]$$

Apply R_{12},

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & -1 & 3 & 2 \\ 5 & -1 & \lambda & \mu \end{array}\right]$$

Apply $R_2 - 2R_1$ and $R_3 - 5R_1$.

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & -6 & \lambda-10 & \mu-10 \end{array}\right]$$

Apply $R_3 + 2R_2$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & \lambda-8 & \mu-6 \end{array}\right]$$

From ①. there can no solution if $\rho(A) \neq \rho(A/B)$
$\Rightarrow$ $\lambda = 8$ and $\mu \neq 6$.

From ①, there can be a unique sol^n if $\rho(A) =$
$\rho(A) = \rho(A/B) =$ no. of variables.
$\Rightarrow \lambda \neq 8$ and $\mu \in \mathbb{R}$

From ①, there can infinite sol^{n}s if
$\rho(A) = \rho(A/B) <$ no. of variables.
$\Rightarrow \lambda = 8$ and $\mu = 6$

Q. Investigate the value of λ and μ so that eq

$x + 2y + 3z = 6$

$x + 3y + 5z = 9$

$2x + 5y + \lambda z = \mu$ ... has

i) no sol^n ii) unique sol^n

iii) infinite sol^n

Ans.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ \mu \end{bmatrix}$$

$A \quad X = B.$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 1 & 3 & 5 & 9 \\ 2 & 5 & \lambda & \mu \end{array}\right]$$

Apply $R_2 - R_1$ and $R_3 - 2R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & \lambda-6 & \mu-12 \end{array}\right]$$

Apply $R_3 - R_2$.

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & \lambda-8 & \mu-15 \end{array}\right] \text{ —— } (1)$$

From (1), there can only be no sol^n if

$\rho(A) \neq \rho(A/B)$

$\Rightarrow \lambda = 8$ and $\mu \neq 15$

From (1), there can only be a unique sol^n if

$\rho(A) = \rho(A/B) =$ no. of variables

$\Rightarrow \lambda \neq 8$ and $\mu \in \mathbb{R}$

From ②, there can only be infinite sol^{n}s if
$\rho(A) = \rho(A/B) <$ no. of variables.
$\Rightarrow \lambda = 8$ and $\mu = 15$.

<u>Homogeneous eq^n ($AX = B$, $B = O$)</u>

1. Write given system in matrix notation $AX = B$.

2. Write augmented matrix

3. Reduce it to echelon form.

4. Find $\rho(A) = \rho(A/B)$

HOMOGENEOUS Eq^n
($AX = B$, $B = O$)
↓
Homogeneous system is always consistent $\rho(A) = \rho(A/B)$

Trivial / Unique sol^n | Non-trivial sol^n

Q. Solve the following system of eq^n.

$x + 2y = 0$

$2x - y + z = 0$

$4x + 3y + 2z = 0$

Ans Given system can be written as where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 1 \\ 4 & 3 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Consider $[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 2 & -1 & 1 & 0 \\ 4 & 3 & 2 & 0 \end{array}\right]$

Apply $R_2 - 2R_1$ and $R_3 - 4R_1$,

$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right]$

Apply $R_3 - R_2$,

$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\rho(A) = 3 = \rho(A/B) = 3 =$ no. of variables.

$\therefore$ system has trivial sol^n.

$x = 0$.

$y = 0$

$z = 0$

Q. Solve the following system of eq^n.

$x + 3y + z = 0$

$2x - 2y - 6z = 0$

$3x + y - 5z = 0$

$$\underbrace{\begin{bmatrix} 1 & 3 & 1 \\ 2 & -2 & -6 \\ 3 & 1 & -5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{B}$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 2 & -2 & -6 & 0 \\ 3 & 1 & -5 & 0 \end{array}\right]$$

Apply $R_2 - 2R_1$ and $R_3 - 3R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -8 & -8 & 0 \\ 0 & -8 & -8 & 0 \end{array}\right]$$

$R_3 - R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -8 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 2 <$ no. of variables.

$\therefore$ system has non-trivial sol^{n}s.

Q. Solve the following system of eq^n.

$2x - y + 3z = 0$

$3x + 2y + z = 0$

$x - 4y + 5z = 0$

Date 7

A

Ans. $\begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$A \quad X = B$

$[A/B] = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 3 & 2 & 1 & 0 \\ 1 & -4 & 5 & 0 \end{array}\right]$

Apply R_{13} and then R_{32}

$[A/B] = \left[\begin{array}{ccc|c} 1 & -4 & 5 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right]$

Apply $R_2 - 2R_1$ and $R_3 - 3R_1$,

$[A/B] = \left[\begin{array}{ccc|c} 1 & -4 & 5 & 0 \\ 0 & 7 & -7 & 0 \\ 0 & 14 & -14 & 0 \end{array}\right]$

Apply $R_3 - 2R_2$

$[A/B] = \left[\begin{array}{ccc|c} 1 & -4 & 5 & 0 \\ 0 & +7 & -7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\rho(A) = \rho(A/B) = 2 <$ no. of variables

$\therefore$ There are non-trivial solution

<u>Homogeneous eq^n containing unknowns.</u>

Investigate for a value of k for the following system of eq^{n}s.

$2x + y + 2z = 0$

$x + y + 3z = 0$

$4x + 3y + kz = 0$ has.

i) trivial solⁿ ii) non-trivial solⁿ

Also find the solution.

Ans.
$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$A \quad X = B$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \\ 4 & 3 & k & 0 \end{array}\right]$$

Apply R_{12}

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & k & 0 \end{array}\right]$$

~~Apply~~ Apply $R_2 - 2R_1$ and $R_3 - 4R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & +4 & 0 \\ 0 & -1 & k-12 & 0 \end{array}\right]$$

Apply $R_3 - R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & k-8 & 0 \end{array}\right]$$

~~ρ(A)…~~ If $k = 8$,

$\rho(A) = \rho(A/B) <$ no. of variables.

They are non-trivial solⁿs.

If $k \neq 8$,

$\rho(A) = \rho(A/B) = 3$ = no. of variables.

∴ There are ~~trav~~ trivial solⁿs.

$x = 0$

$y = 0$

$z = 0$.

Q. Investigating for a value of k for the follow system of eqⁿs.

$x + 2y - z = 0$

$3x + (k+7)y - 3z = 0$

$2x + 4y + (k-3)z = 0$

i) trivial solⁿ ii) non-trivial solⁿ.

Ans.
$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & k+7 & -3 \\ 2 & 4 & k-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = B.

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 3 & k+7 & -3 & 0 \\ 2 & 4 & k-3 & 0 \end{array}\right]$$

Apply R_{23},

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 4 & k-3 & 0 \\ 3 & k+7 & -3 & 0 \end{array}\right]$$

Apply $R_2 - 2R_1$ and $R_3 - 3R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 0 & k-1 & 0 \\ 0 & k+1 & 0 & 0 \end{array}\right]$$

Here $C_K = R_{23}$,

$$[A/B] = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & k+1 & 0 & 0 \\ 0 & 0 & k-1 & 0 \end{bmatrix}$$

If $k = 1$,

$\rho(A) = 2 = \rho(A/B) <$ no. of variables,

$\therefore$ There will non-trivial sol^{n}s.

If $k \neq 1$,

$\rho(A) = \rho(A/B) = 3 =$ no. of variables.

$\therefore$ There will be trivial sol^{n}s.

Vectors

Linear independent vectors:

3 vectors $x_1, x_2, x_3, \ldots, x_n$ are said to be linearly independent, if every relationship of the type $C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_nX_n = 0$ implies $C_1 = C_2 = C_3 = \ldots = C_n = 0$ when $C_1, C_2, C_3, \ldots, C_n$ are a non-zero scalar, and the O is a null matrix.

Linear dependent vectors:

If at least one $C_i \neq 0$ for $i = 1, 2, 3, \ldots, n$, then the vectors are said to be linearly dependent

To check dependency of vectors

Step I: Write down given vector $X_1, X_2, X_3, \ldots, X_n$

Step II: Consider matrix eq^n.

$$C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_nX_n = 0$$

Step III: Write it in the ~~most~~ matrix form $AX = B$.

Step IV: Consider augmented matrix $[A/B]$

Step V: Reduce $[A/B]$ to equivalent form

Step VI: Find $\rho(A)$ and $\rho(A/B)$

Step VII: To decide the nature of the sol^n of the system, use chart (homogeneous chart)

Step VIII :

a) If system has trivial sol^n $(C_1 = C_2 = C_3 = \cdots = C_n = 0)$ then $x_1, x_2, x_3, \ldots, x_n$ are linearly independent.

b) If system has non-trivial sol^n. $(C_i \neq 0$ for $i = 1, 2, \ldots, n)$ then $x_1 = x_2 = \cdots = x_n$ are linearly dependent.

Step IX : If given vector = linearly ~~independent~~ then find relation between them.

Example Examine for linearly dependenty. If dependent, find the relation between them.

1) $(2, -1, 3, 2)(1, 3, 4, 2), (3, -5, 2, 2)$

Ans. $x_1 = (2, -1, 3, 2)$

$x_2 = (1, 3, 4, 2)$

$x_3 = (3, -5, 2, 2)$

Consider $C_1 x_1 + C_2 x_2 + C_3 x_3 = 0$ —— ①

$C_1(2, -1, 3, 2) + C_2(1, 3, 4, 2) + C_3(3, -5, 2, 2) = 0$

$$\Rightarrow 2C_1 + C_2 + 3C_3 = 0$$
$$-C_1 + 3C_2 - 5C_3 = 0$$
$$3C_1 + 4C_2 + 2C_3 = 0$$
$$2C_1 + 2C_2 + 2C_3 = 0$$

which is homogeneous system at eq^n $AC = B$

where $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 3 & -5 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ -1 & 3 & -5 & 0 \\ 3 & 4 & 2 & 0 \\ 2 & 2 & 2 & 0 \end{array}\right]$$

R_{21} and R_{34},

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 2 & 1 & 3 & 0 \\ 2 & 2 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$$

$R_2 - 2R_1$ and $R_3 - 2R_1$ and $R_4 - 3R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 0 & 7 & -7 & 0 \\ 0 & 13 & -13 & 0 \\ 0 & 8 & -8 & 0 \end{array}\right]$$

$\frac{1}{7}R_2$, $\frac{1}{13}R_3$, and $\frac{1}{8}R_4$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$R_3 - R_2$, $R_4 - R_2$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow \rho(A) = \rho(A/B) = 2 <$ no. of variables $= 3$

$\therefore$ the system has non-trivial sol^{n}s.

Put $C_3 = t,\quad t \in R$

From R_2,

$C_2 - C_3 = 0$

$C_2 = t$

From R_1,

$C_1 - 3C_2 + 5C_3 = 0$

$C_1 - 3t + 5t = 0$

$\cancel{C_1 - 8t = 0}$

$\cancel{C_1 = -8t}$

$C_1 + 2t = 0$

$C_1 = -2t$

∴ Solⁿ set: $C_1 = -2t$

$C_2 = t$

$C_3 = t \quad [t \in R]$

Note that C_1, C_2, C_3 are non-zero

∴ x_1, x_2, x_3 are linearly dependent

From 2,

$C_1x_1 + C_2x_2 + C_3x_3 = 0$

$-2t\,x_1 + t\,x_2 + t\,x_3 = 0$

$t(-2x_1 + x_2 + x_3) = 0$

$-2x_1 + x_2 + x_3 = 0$

$x_2 + x_3 = 2x_1$

Date

Example $(1,-1,-1)$, $(2,1,1)$, $(3,0,2)$,

Ans. $X_1 = (1,-1,-1)$

$X_2 = (2,1,1)$

$X_3 = (3,0,2)$

$C_1X_1 + C_2X_2 + C_3X_3 = 0$

$C_1(1,-1,-1) + C_2(2,1,1) + C_3(3,0,2) = 0$

$C_1 + 2C_2 + 3C_3 = 0$

$-C_1 + C_2 + 0 = 0$

$-C_1 + C_2 + 2C_3 = 0$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 2 & 0 \end{array}\right]$$

Apply R_{23},

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -1 & 1 & 2 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right]$$

Apply $R_1 + R_2$ and $R_1 + R_3$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right]$$

Apply $R_3 - R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 3 =$ no. of variables.
$\therefore$ There are trivial sol^{n}s.

$-2C_3 = 0$
$C_3 = 0$.

$\therefore 3C_2 + 5C_3 = 0$
$3C_2 = 0$
$C_2 = 0$

$C_1 + 2C_2 + 3C_3 = 0$
$C_1 = 0$.

$\therefore$ They are linearly independent matrix.

Example. $(1, 1, 1, 3)$, $(1, 2, 3, 4)$, $(2, 3, 4, 7)$

$X_1 = (1, 1, 1, 3)$
$X_2 = (1, 2, 3, 4)$
$X_3 = (2, 3, 4, 7)$

$C_1X_1 + C_2X_2 + C_3X_3 = 0$
$C_1(1, 1, 1, 3) + C_2(1, 2, 3, 4) + C_3(2, 3, 4, 7) = 0$

$C_1 + C_2 + 2C_3 = 0$
$C_1 + 2C_2 + 3C_3 = 0$
$C_1 + 3C_2 + 4C_3 = 0$
$3C_1 + 4C_2 + 7C_3 = 0$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \\ 3 & 4 & 7 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 4 & 0 \\ 3 & 4 & 7 & 0 \end{array}\right]$$

Apply $R_2 - R_1$, $R_3 - R_1$ and $R_4 - 3R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

Apply $R_3 - R_2$ and $R_4 - R_3$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 2 <$ no. of variables

$\therefore$ there are non-trivial soln s.

$C_2 + C_3 = 0$ —— ①

$C_2 = -C_3$

$C_1 + (C_2 + C_3) = 0$

$C_1 + 0 = 0$ [From ①]

$C_1 = 0$

$\therefore C_1 = 0$

$C_2 = -C_3$

They are linearly dependent.

5) $(1,2,-1,0)$, $(1,3,1,2)$, $(4,2,1,0)$, $(6,1,0,1)$

Ans. $x_1 = (1,2,-1,0)$

$x_2 = (1,3,1,2)$

$x_3 = (4,2,1,0)$

$x_4 = (6,1,0,1)$

$C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 = 0$

$C_1(1,2,-1,0) + C_2(1,3,1,2) + C_3(4,2,1,0) + C_4(6,1,0,1) = 0$

$C_1 + C_2 + 4C_3 + 6C_4 = 0$

$2C_1 + 3C_2 + C_3 + C_4 = 0$

$-C_1 + C_2 + C_3 + 0C_4 = 0$

$0C_1 + 2C_2 + 0C_3 + C_4 = 0$

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 2 & 3 & 1 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$A \quad X = B$

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 4 & 0 \end{array}\right]$$

Apply R_{23},

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ 0 & 2 & 0 & 4 & 0 \end{array}\right]$$

Apply $R_2 + R_1$ and $R_3 - 2R_1$,

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 0 & 2 & 5 & 6 & 0 \\ 0 & 1 & -7 & -11 & 0 \\ 0 & 2 & 0 & 4 & 0 \end{array}\right]$$

Apply ~~$R_3 - R_4$~~ $R_2 - 2R_3$, $R_4 - R_2$

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 0 & 2 & 5 & 6 & 0 \\ 0 & 0 & 19 & 28 & 0 \\ 0 & 0 & -5 & -2 & 0 \end{array}\right] \begin{array}{c} \\ \\ 5 \\ 19 \end{array}$$

~~Apply $R_4 \times R$,~~

~~$[A/B] =$~~

Apply $5R_3 + 19R_4$,

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 6 & 0 \\ 0 & 2 & 5 & 6 & 0 \\ 0 & 0 & 19 & 28 & 0 \\ 0 & 0 & 0 & 110 & 0 \end{array}\right]$$

$\rho(A) = \rho(A/B) =$ no. of variables

$\therefore$ There are non-trivial questions.

$110\, C_4 = 0$

$C_4 = 0$

$2C_2 = 0$

$C_2 = 0$

$19C_3 + 28C_4 = 0$

$19C_3 = 0$

$C_3 = 0$

$C_1 + C_2 + 4C_3 + 6C_4 = 0$

$C_1 + 0 + 0 + 0 = 0$

$C_1 = 0$

$2C_2 + 5C_3 + 6C_4 = 0$

$C_1 = C_2 = C_3 = C_4 = 0$

linearly independent.

Applications of System of linear eq^n.

1. Electric current

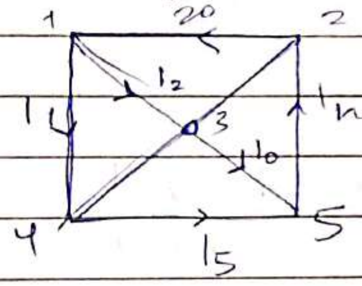
→ ohm's law - the voltage drop (V) across a resist-
- ance (R) is $V = IR$ where V is

2. → KCL (Kirchoff's current law)

The algebraic sum of the current entering any node is 0. OR
The current flow into a node = current flow out of the node

→ Kirchoff's voltage dro
The ~~averar~~ algebraic sum of the voltage drop around a closed loop = total voltage ~~loop~~. source in the node.

Q. Find the ~~bra~~ current in the various branches of the following network.



By KCL Law,
In current = out current, at any node

At node 1, $I_2 + I_2 = 20 \Rightarrow I_2 + I_2 = 20$

At node 2, $I_4 = I_3 + 20 \Rightarrow -I_3 + I_4 = 20$

At node 3, $I_2 + I_3 = 10 + 10 \Rightarrow I_2 + I_3 = 20$

At node 4, $I_1 + 10 = I_5 \Rightarrow I_1 - I_5 = -10$

At node 5, $I_5 + I_{10} = I_4 \Rightarrow I_4 - I_5 = 10$

$$[A/B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & +1 & -1 & 10 \end{array}\right]$$

Apply $R_4 - R_1$

$$[A/B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 0 & -1 & -30 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

Apply R_{23}

$$[A/B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & -1 & 0 & 0 & -1 & -30 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

Apply $R_4 \to R_4 + R_2$,

$$[A/B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 1 & 0 & -1 & \cancel{50} -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

$R_3 + R_4$.

$$[A/B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{array}\right]$$

Apply $R_5 - R_4$,

$$[A/B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 4 <$ no. of variables.

$\therefore$ System is ~~consistent~~ ~~trivial~~ consistent

$\therefore$ There are infinite sol^{n}s.

$I_4 - I_5 = 10$ [Let $I_5 = t$]

$I_4 = 10 + I_5$

$I_4 = 10 + t$

$I_5 = t$

$\cancel{R_3}$ $-I_3 + I_4 = 20$

$-I_3 + 10 + t = 20$

$t - I_3 = 10$

$I_3 = t - 10$

$I_2 + I_3 = 20$

$I_2 + t - 10 = 20$

$I_2 + t = 30$

$I_2 = 30 - t$

$I_1 + I_2 = 20$

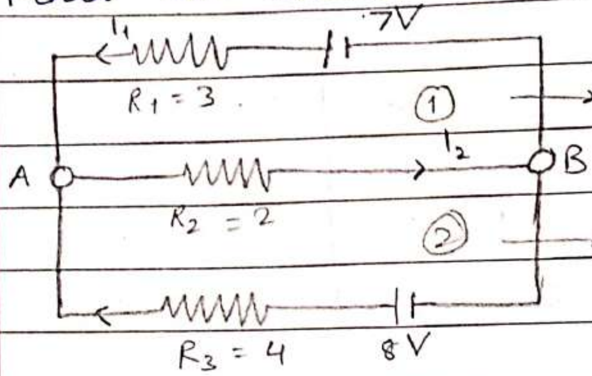
$I_1 + 30 - t = 20$

$I_2 = -10 + t$

$I_2 = t - 10$

$\therefore$ It is linearly dependent.

Q. Find the current in the ~~ckt~~ circuit shown.



By KCL's law,

$I_1 + I_3 = I_2$

$I_2 = I_1 + I_3$

$\Rightarrow I_1 - I_2 + I_3 = 0$

By KVL law,

ΣV = total voltage

For upper loop ①,

$I_1 R_1 + I_2 R_2 = 7$

$I_3 R_3 + I_2 R_2 = 8$ [For lower loop ②]

$I_2 R_2 = 7 - I_1 R_1 = 8 - I_3 R_3$

$I_3 R_3 - I_1 R_1 = 1$

$I_1 - I_2 + I_3 = 0$

$3I_1 + 2I_2 = 7$

$2I_2 + 4I_3 = 8$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 3 & 2 & 0 & 7 \\ 0 & 2 & 4 & 8 \end{array}\right]$$

Apply $R_2 - 3R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 5 & -3 & 7 \\ 0 & 2 & 4 & 8 \end{array}\right]$$

Apply $5R_3 + 2R_1$

~~$[A/B] = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & 0 & 7 \end{bmatrix}$~~

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 5 & -3 & 7 \\ 0 & 0 & 26 & 26 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 3 =$ no. of variables

$\therefore$ the system is consistent and it has unique sol^n.

$26 I_3 = 26$

$I_3 = 1$

$5I_2 - 3I_3 = 7$

$5I_2 - 3 = 7$

$I_2 = 2$

$I_1 - I_2 + I_3 = 0$

$I_2 - 2 + 1 = 0$

$I_2 - 1 = 0$

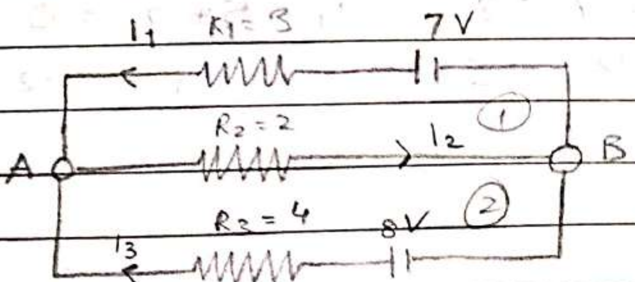
$I_1 = 1.$

$\therefore I_1 = 1.$

$I_2 = 2$

$I_3 = 1.$

Q. Find the current in the circuit



By KCL law,

$I_1 + I_3 = I_2$

$I_2 = I_1 + I_3$

$\Rightarrow I_1 - I_2 + I_3 = 0$

By KVL law,

$\Sigma V =$ total voltage

$I_1 R_1 + I_2 R_2 = 7$

$I_2R_2 + I_3R_3 = 8$

$I_1R_1 + I_2R_2 = 7$

$I_1 - I_2 + I_3 = 0$

$3I_2 - I_3 = 8$

$4I_1 + 3I_2 = 7$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 8 \\ 4 & 3 & 0 & 7 \end{array}\right] = [A/B]$$

Apply $R_3 - 4R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 8 \\ 0 & 7 & -4 & 7 \end{array}\right]$$

7

3 $\quad -12+7 \quad \frac{56}{21}$

35

Apply $3R_3 - 7R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 8 \\ 0 & 0 & -5 & 35 \end{array}\right]$$

∴ the system is consistent and has unique solⁿ.

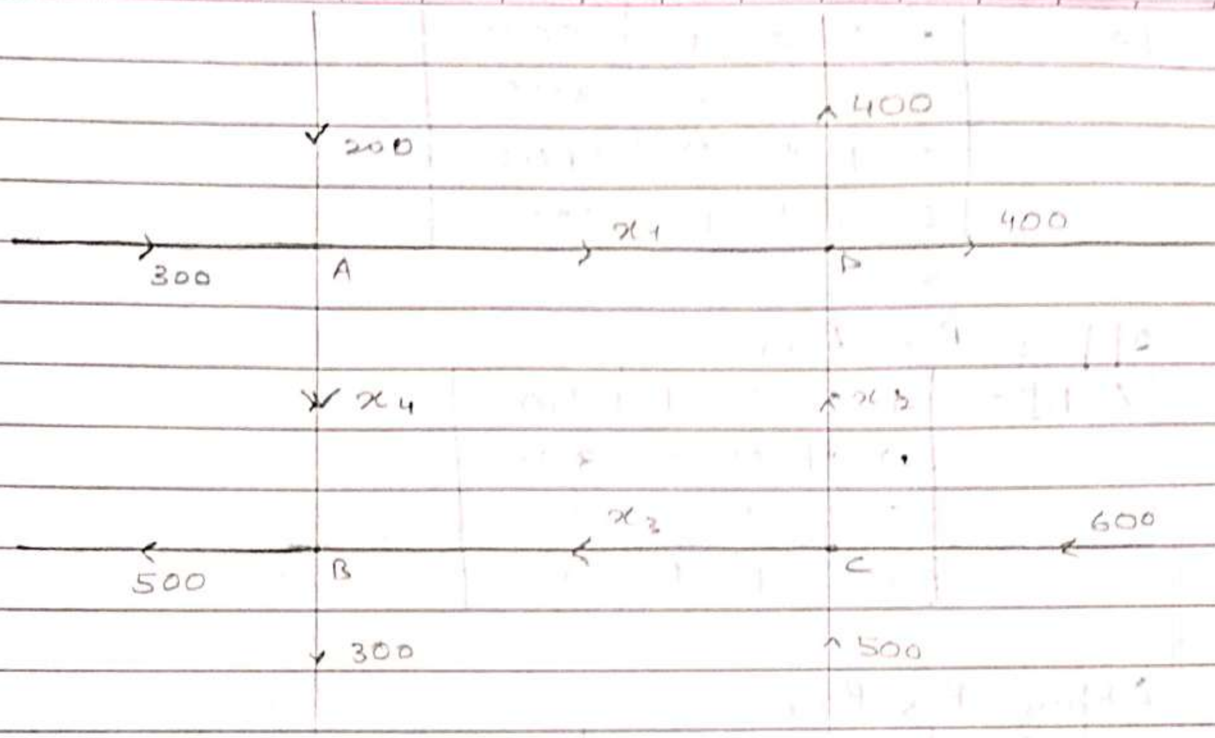
$-5I_3 = -35$ $\qquad$ $3I_2 - I_3 = 8$ $\qquad$ $I_1 - I_2 + I_3 = 0$

$I_3 = 7$ $\qquad$ $3I_2 - 7 = 8$ $\qquad$ $I_1 - 5 + 7 = 0$

$I_2 = 5$ $\qquad$ $I_1 = -2.$

Q. Solve the following graph problem

Q.



$x_1 + x_4 = 300 + 200 = 500$

$x_3 + x_4 = 300 + 500 = 800$

$x_2 + x_3 = 1100$

$x_1 + x_2 = 800$

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 500 \\ 800 \\ 1100 \\ 800 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 500 \\ 0 & 0 & 1 & 1 & 800 \\ 0 & 1 & 1 & 0 & 1100 \\ 1 & 1 & 0 & 0 & 800 \end{array}\right]$$

Apply R_{24} and then R

Date

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 500 \\ 1 & 1 & 0 & 0 & 800 \\ 0 & 1 & 1 & 0 & 1100 \\ 0 & 0 & 1 & 1 & 800 \end{array}\right]$$

Apply $R_2 - R_1$,

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 500 \\ 0 & +1 & 0 & -1 & 300 \\ 0 & 1 & 1 & 0 & 1100 \\ 0 & 0 & 1 & 1 & 800 \end{array}\right]$$

Apply $R_3 - R_2$,

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 500 \\ 0 & 1 & 0 & -1 & 300 \\ 0 & 0 & 1 & 1 & 800 \\ 0 & 0 & 1 & 1 & 800 \end{array}\right]$$

Apply $R_4 - R_3$,

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 500 \\ 0 & 1 & 0 & -1 & 300 \\ 0 & 0 & 1 & 1 & 800 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 3 <$ no. of variables.

From (3): $x_3 + x_4 = 800$ $\quad\quad x_4 = t$

$x_3 = 800 - t$

$x_2 - x_4 = 300$ $\quad\quad x_1 + x_4 = 500$

$x_2 = 300 + t$ $\quad\quad x_1 = 500 - t$

∴ The system is consistent and has unique solⁿ.

Linear transformation

Represented by :

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = y_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = y_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_3 = y_3$

$\vdots$

$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n = y_n$

• In matrix notation, it can be written as,

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & \vdots & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_3 \end{bmatrix}$$

$$A \quad X = Y$$

Also the linear transformation from n variables to the variable $y_1, y_2, y_3, \dots, y_n$.
i.e. the linear transformation of the vector X to the vector Y.

Types of linear transformation

1. Inverse

• For a non-singular transformation $Y = X$ since 'A' is not singular i.e. $(|A| \neq 0)$; A^{-1} exists and we can write the inverse transformation $X = A^{-1}Y$.

• ~~method~~ Step I
→ write given linear transformation $Y = X$

• Step II

Page No.
Date

- find ~~the~~ $|A|$
  → if $|A| \neq 0$, the system possesses unique solⁿ.

- ~~Find~~ Step III : Find A^{-1}
  → $A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$

- Step IV : solve $X = A^{-1}Y$
  ~~→ given the transformation Y=~~

Q. Given the transformation $Y = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

Find the coordinate x_1, x_2, x_3 in X corresponding to $(1, 2, -1)$ in Y.

Ans. $AX = Y$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

~~$|A| = 2(2-0) - 1(0) + 1(0-1) = 4 - 0 - 1 = 3$~~
~~$= 2 \times 2 - 0 + 1 = 3$~~

$|A| = 2(-2-0) - 1(-2-2) + 1(0-1) = -4 + 4 - 1 = -1 \neq 0$

∴ The system has unique solⁿs.

$x_1 - 2x_3 = -1$ $\qquad x_3 = t$

$x_1 - 2t = -1$

$x_1 = 2t - 1$

$$x_1 + x_2 + 2x_3 = 2$$
$$(2t-1) + x_2 + 2t = 2$$
$$2t - 1 + x_2 + 2t = 2$$
$$4t + x_2 = 3$$
$$x_2 = 3 - 4t$$

$$2x_1 + x_2 + x_3 = 1$$
$$2(2t-1) + (3-4t) + x_3 = 1$$
$$\cancel{4t} - 2 + 3 - \cancel{4t} + x_3 = 1$$
$$x_3 \cdot$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj} A$$

$$(-1)^3$$

$$= (-1) \begin{bmatrix} -2 & +2 & -1 \\ 4 & -5 & +1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 1 \\ 4 & -5 & -3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix}$$

$$X = A^{-1} Y$$

$$= \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

Q. The transformation from $X = Y$. is given by $Y = X$. Another transformation from $Y = Z$ is given $Z = Y$: the composite transformation from $X = Z$ is given by:

Ans. $Z = BY$

$Z = B(AX) = (BA)X$

Q. Express each transformation $x_1 = 3y_1 + 2y_2$

$x_2 = -y_1 + 4y_2$

and $y_1 = z_1 + 2z_2$

$y_2 = 3z_1$ in the matrix form. and find the composite transformation which expresses (x_1, x_2) in terms of (z_1, z_2).

Ans.

Q. $Y = \begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Find the coordinate (x_1, x_2, x_3) corresponding to $(2, 9, 5)$.

$(2, 1, -1)$

Eigen Values and Eigen vectors

Let A be a square matrix

X be a column matrix

λ be a scalar

$AX = \lambda X$

$AX - \lambda I X = 0$ where I is a .

$(A - \lambda I) X = B = 0$

- if $X \neq 0$,

- root of ~~the~~ $|A - \lambda I| = 0$ is Eigen's value if $X = ($

$$\underbrace{(A - \lambda I)}_{\text{Eigen value}} \overbrace{X}^{\text{Eigen vector}} = 0$$

UNIT: 2

Eigen values and Eigen vectors

Let A be $n \times n$ square matrix,
Y and X be non-zero column vector ~~so~~ such that $Y = AX$ (linear transformation)

Consider the transformation for a given matrix A which gives $Y = \ldots$

$Y = \lambda X$

$AX = \lambda X$

$AX = \lambda I X$

$AX - \lambda I X = 0$

$\therefore (A - \lambda I) X = 0$ where [$(A - \lambda I)$ = eigen value
X = eigen vector]

$$\begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33}-\lambda & \cdots & a_{3n} \\ & & \vdots & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn}-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

This is a homogeneous system of linear eq^n. It will have a non-trivial sol^n ($X \neq 0$). If ~~determinant~~ $|A - \lambda I| = 0$, if A is a square matrix then the characteristic polynomial of matrix A is $|A - \lambda I|$ and characteristic eq^n $|A - \lambda I| = 0$

The root of the characteristic eq^n are known as

Eigen values.

Eigen vectors

For every characteristic root, $(A - \lambda I)x = 0$ will have a non-trivial sol^n.

Let X_i with a sol^n vector $(A - \lambda I)X = 0$ correspo-nding to the characteristic root λ_i, then X_i is called the eigen vector associated with the root λ.

· Characteristic eq^n (2×2)

Let A be a matrix of order 2, then the chara-cteristic eq^n of A is

$\lambda^2 - S_1\lambda + |A| = 0$ where.

S_1 = sum of diagonal elements.

$= [a_{11} + a_{22}]$ = Trace of A

$\lambda^2 - A\lambda + B = 0$

$A = S_1$

B = determinant

Characteristic eq^n (3×3)

Let A be a matrix of order 3, then the characteri-stic eq^n of A is

$\lambda^3 - S_1\lambda^2 + S_2\lambda + |A| = 0$

· $S_1 = [a_{11} + a_{22} + a_{33}]$ = Trace of A

· S_2 = ~~sum of~~ minor(a_{11}) + minor(a_{22}) + minor(a_{33})

~~Eigen val~~

Q. Find the eigen value of $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$. The

characteristic eqn of A is $|A - \lambda I| = 0$ and

$$\begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

Ans. $\lambda^2 - S_1 \lambda + |A| = 0$

$\lambda^2 - 10\lambda + |A| = 0$

$|A| = \cancel{(8-\lambda)}\ 16 + 8 = 24$

$\lambda^2 - 10\lambda + 24 = 0$

$\lambda^2 - 6\lambda - 4\lambda + 24 = 0$

$\lambda(\lambda - 6) - 4(\lambda - 6) = 0$

$(\lambda - 6)(\lambda - 4) = 0$.

Page No.
Date

Q. Type 1 : Matrix is non-symmetric, eigen values are non-repeated.

Ans. $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

→ The characteristic eq^n of A is

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$$

i.e. $\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

where $S_1 = \text{trace} = 1+2+3 = 6$

S_2 = minor of a_{11} + minor of a_{22} + minor of a

$= \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$

$S_2 = 4 + 5 + 2 = 11$

$|A| = 6$

$\therefore \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda = 1, 2, 3$

To find eigen vector $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ corresponding t

i.e. eigen values $\lambda = 1, 2, 3$.

$0x + 0y - z = 0$

$x + y + z = 0$

$2x + 2y + 2z = 0.$

$$x = \begin{bmatrix} t \\ -t \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = t$$

Q. Eigen vector $x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$x = 1$

Ans $-x + 0y - z = 0$

$x + 0y + z = 0$

$2x + 2y + z = 0$

$$\frac{x}{\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix}} = t$$

Qx $x = -2t$

$y = t$

$z = 2t$

$$x_2 = \begin{bmatrix} -2t \\ t \\ 2t \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Q. Find the eigen value of matrix and eigen vector of the following.

Ans.

Date 30 9 23

~~Q. Find the eigen~~

Q. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & 3 \end{bmatrix} = 1(15) - 2(0) + 3(0) = 15 - 0 + 0$

$15 + 3 + 5 = 23$

Apply $2R_2 - 5R_1$,

$A = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 5-\lambda & 7 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| =$

$S_1 = 9$

$S_2 = (15) + (0) + (5)$

$= ~~10~~ \; 20$

$15^2 = 1225$

$15^3 = 5375$

$A = \begin{bmatrix} 5 & 0 & -1 \\ 0 & 5 & 7 \\ 0 & 0 & 3 \end{bmatrix}$

$\lambda^3 - 9\lambda^2 + 23\lambda - 15 = 0$ $\quad A^2$

$\lambda = 1, 5, 3$

$P =$

Q. Find the eigen value of matrix A =

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

. Also find eigen vector corresponding to largest diagonal.

Ans.

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix} \qquad A = \begin{bmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{bmatrix} = 0$$

Apply R_{21} and then R_{23}

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & -4 & -3 \\ 4 & 6 & 6 \end{bmatrix}$$

~~$\lambda = 4, 3, 3$~~

~~Apply $2R_2 - 4R_1$~~

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

~~Apply~~

~~SG1~~

Apply $R_3 - 4R_1$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & -4 & -3 \\ 0 & -6 & -2 \end{bmatrix}$$

Apply $R_2 + R_1$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & -1 \\ 0 & -6 & -2 \end{bmatrix}$$

Apply ~~$R_3 + 2R_2$~~ $R_3 - 6R_2$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

For $\lambda = 4$,

$$\begin{bmatrix} 0 & 6 & 6 \\ 1 & -1 & 2 \\ -1 & -4 & -7 \end{bmatrix}$$

$$|A| = 1(-4) - 3(0) + 2(0)$$
$$= -4$$

$$\therefore \lambda = (1, -1, 4)$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

$$S_1 = 1 - 1 + 4 = 4$$
$$S_2 = (-4) + (4) + (-1)$$
$$= -1$$

$$\lambda^3 - 4\lambda^2 - 1\lambda + 4 = 0$$

For $\lambda = 4$

$$\begin{bmatrix} 0 & 6 & 6 \\ 1 & -1 & 2 \\ -1 & -4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x + 6y + 6z = 0$$
$$x - y + 2z = 0$$
$$-x - 4y - 7z = 0$$

$$\frac{x}{\begin{vmatrix} -1 & 2 \\ -4 & -7 \end{vmatrix}}$$

Date

Q. Find the module matrix with diagonalised $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$.

Ans.

$$A = \begin{bmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{bmatrix} = 0$$

Apply $5R_2 - 3R_1$,

$$A = \begin{bmatrix} 5 & 3 \\ 0 & 16 \end{bmatrix}$$

Characteristic eqn,

$$|A - \lambda I| = 0$$

$$A = \begin{bmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - S_1\lambda + |A| = 0$$

$$S_1 = 5 + 16 = 21$$

$$\lambda^2 - \overset{10}{\cancel{21}}\lambda + 16 = 0$$

For $\lambda = 8$

$$\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x + 3y = 0$$

$$-x + y = 0$$

$$x = y$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For $\lambda = 2$,

$$\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3x + 3y = 0$

$3x + 3y = 0.$

$x = -y$

$$X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Date

Q. $2x - y + 3z = 4$ | Eigen values.

$x + y - 3z = -1$

$15x - 3y + 9z = 21.$

Ans.

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & -3 \\ 15 & -3 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 21 \end{bmatrix}$$

$$A \quad X = B$$

~~Apply~~

$$A = \begin{bmatrix} 2-\lambda & -1 & 3 \\ 1 & 1-\lambda & -3 \\ 15 & -3 & 9-\lambda \end{bmatrix}$$

Apply R_{21}

$$A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & -1 & 3 \\ 15 & -3 & 9 \end{bmatrix}$$

Apply $R_2 - 2R_1$,

$$A = \begin{bmatrix} 1 & 1 & -3 \\ 0 & -3 & 9 \\ 15 & -3 & 9 \end{bmatrix}$$

Apply $R_3 - 15R_1$

$$A = \begin{bmatrix} 1 & 1 & -3 \\ 0 & -3 & 9 \\ 0 & -18 & 54 \end{bmatrix}$$

Apply $R_3 - 6R_2$

~~Apply~~ $$A = \begin{bmatrix} 1 & 1 & -3 \\ 0 & -3 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$

Page No.
Date

$$A = \begin{bmatrix} 1 & 1 & -3 \\ 0 & -3 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$

$\lambda = 1, -3, 0$

For $\lambda = 1$,

$$A = \begin{bmatrix} 0 & -1 & -3 \\ 0 & -4 & 9 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & -3 \\ 15 & -3 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 21 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

Apply R_{12},

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 2 & -1 & 3 & 4 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

Apply $R_2 - 2R_1$ and $R_3 - 15R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -3 & 9 & 6 \\ 0 & -18 & 54 & 36 \end{array}\right]$$

-3×-6

Apply ~~$R_3 - 6R_2$~~,
(NEXT)

$-18 + (18) = 0$

$-18 + (-3 \times -6) = 0$

$R_3 + (R_2 \times -6) = 0$

$(R_3 - 6R_2) = 0$

Apply $R_3 - 6R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -3 & 9 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore \rho(A) = \rho(A/B) = 2 <$ no. of variable.

$\therefore$ The sol^{n}s are infinite

Q. Solve values of λ and μ. so that the eq^n

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$ has

i) no sol^n ii) unique sol^n

iii) infinite sol^{n}s.

Ans.

UNIT : 2

Eigen Values and Eigen Vectors

Let A be a $n \times n$ square matrix, Y and X are non-zero column vector such that $Y = AX$ (linear transformation)

Consider the tranformation for a given matrix A which gives $Y = \lambda X$.

$\therefore AX = \lambda X$

$AX = \lambda I X$

$AX - \lambda I X = 0$

$X(A - \lambda I) = 0$

$$\begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33}-\lambda & \cdots & a_{3n} \\ & & \vdots & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn}-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

This is a homogeneous system of linear eq^n.
$\Rightarrow$ It will have a non-trivial sol^n. ($x \neq 0$) if
$|A - \lambda I| = 0$

The roots of characteristic eq^n are known as eigen values.

For every characteristic rule $(A - \lambda I)X = 0$ will have a non-trivial sol^n.

$$(A - \lambda I)X = 0$$

where

- the eq^n is homogeneous.
- λ is eigen value
- X is eigen vector

→ If all eigen vectors are equal, they are symmetric.

- eigen vectors lin. independ. → trivial

eigen vectors lin. depend. → non-trivial

Let X_1 with a solution vector $(A-\lambda I)X = 0$ corresponding to the characteristic value λ_1, then X_1 is called eigen vector associated with the value λ_1.

Characteristic Eqⁿ

1. Let A be a matrix of order 2, then the characteristic eqⁿ of A is:
   - $\lambda^2 - S_1\lambda + |A| = 0$ where
   - S_1 = sum of diagonals

2. Let A be a matrix of order 3, then the characteristic eqⁿ of A is:
   - $\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$
   - S_1 = sum of diagonals

   - S_2 = sum of minors diagonals
     = $min(a_{11}) + min(a_{22}) + min(a_{33})$

Q. Find eigen value of $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.

Ans. Characteristic eqⁿ of A is
$(A - \lambda I)X = 0$

$A = \begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix}$

$\lambda^2 - S_1\lambda + |A| = 0$

$S_1 = 8+2 = 10$

$|A| = 16+8 = 24$

$\lambda^2 - 10\lambda + 24$

$\lambda = 6, 4$

$\therefore$ Eigen values of A are 4 and 6.

Properties of eigen values

1. The sum of the eigen values of the matrix is the sum of the element of the principal diagonal.

$\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n = a_{11} + a_{22} + a_{33} + \dots + a_{nn} =$ Trace

2. The product of the eigen value of a matrix A is equal to its determinant.

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \dots \cdot \lambda_n = |A|$

3. The eigen value of an upper triangular, and lower triangular or diagonal matrix are the diagonals.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 9 \\ 0 & 0 & -7 \end{bmatrix}$ (Upper Triangular Matrix)

• eigen value of A are 1, 4, -7

4. Matrix A is non singular i.e. $|A| \neq 0$ if and only if $\lambda_j \neq 0$
$j = 1, 2, 3, \dots, n$

5. A matrix A is singular if and only if it has at least 1 eigen value which is zero.

Page No.
Date

Properties of eigen vectors

1. Eigen vector is never zero.

2. Eigen vector corresponding to distant eigen values always linearly dependent.

Type 1: Non-symmetrical matrix, eigen values are non repeated.

Q. Find eigen value and eigen vector of following matrix.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

Ans. Characteristic eqn of A

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad \text{—— (1)}$$

$$A = \begin{bmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{bmatrix} = 0$$

$S_1 = 1 + 2 + 3 = 6$ —— (2)

$$S_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$$

$= 4 + 5 + 2$

$= 11$ —— (3)

$|A| = 1 \cdot 2 \cdot 3 = 6$ —— (4)

Substituting (2), (3), and (4) in (1).

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda^2(\lambda$

Page No.
Date

$\therefore$ The eigen values are 1, 2, 3 (Non-repeated)

To find eigen vector $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ corresponding to eigen values $\lambda = 1, 2, 3.$

Consider $(A - \lambda I)X = 0$

$$\begin{bmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = 1$,

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

~~$-z = 0$~~
~~$x + y + z = 0$~~
~~$2x + 2y + 2z = 0$~~

$0x + 0y - z = 0$
$x + y + z = 0$
$2x + 2y + 2z = 0$

By Cramer's Rule, (1st and 2nd)

$$\frac{x}{\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}} = t$$

$$\frac{x}{1} = \frac{-y}{1} = \frac{z}{0} = t$$

$x = t$
$y = -t$
$z = 0$

Date

$$X_1 = \begin{bmatrix} t \\ -t \\ 0 \end{bmatrix} = t \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

$$X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

For $\lambda = 2$,

$$\begin{bmatrix} 1-2 & 0 & -1 \\ 1 & 2-2 & 1 \\ 2 & 2 & 3-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x + 0y - z = 0$$
$$x + 0y + z = 0$$
$$2x + 2y + z = 0$$

By Cramer's Rule (2nd and 3rd)

$$\frac{x}{\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix}} = t$$

$$\frac{x}{-2} = \frac{-y}{-1} = \frac{z}{2} = t$$

$$\therefore x = -2t$$
$$y = t$$
$$z = 2t$$

$$x_2 = \begin{bmatrix} -2t \\ t \\ 2t \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

For $\lambda = 3$,

$$\begin{bmatrix} 1-3 & 0 & -1 \\ 1 & 2-3 & 1 \\ 2 & 2 & 3-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x + 0y - z = 0$

$x - y + z = 0$

$2x + 2y + 0z = 0$

By Cramer's Rule (1st and 2nd),

$$\frac{x}{\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}} = t$$

$$\frac{x}{-1} = \frac{-y}{-1} = \frac{z}{2} = t$$

$\therefore x = -t$

$y = t$

$z = 2t$

$$X_3 = \begin{bmatrix} -1 \\ 1 \\ 21 \end{bmatrix} = 1 \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ Matrix is not symmetric

$\therefore$ Eigen vector X_1, X_2, X_3 are linearly independent.

Q. Find eigen value and eigen vector of following matrix

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 2 & 0 & -3 \end{bmatrix}$$

Ans. The characteristic eqn of A is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

$$S_1 = -1 - 2 - 3 = -6$$

$$S_2 = \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} -1 & 1 \\ 0 & -2 \end{vmatrix}$$

$$= 6 - 1 + 2$$

$$= 7$$

$$|A| = -1(6) - 1(-2) + 2(4)$$

$$= -6 + 2 + 8 = 4$$

$$\lambda^3 + 6\lambda^2 + 7\lambda - 4 = 0$$

$$\lambda = -4, \; 0.414, \; -2.414$$

For $\lambda = 4$,

$$A = \begin{bmatrix} -1-4 & 1 & 2 \\ 0 & -2-4 & 1 \\ 2 & 0 & -3-4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 1 & 2 \\ 0 & -6 & 1 \\ 2 & 0 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-5x + y + 2z = 0$$
$$0x - 6y + z = 0$$
$$2x + 0y - 7z = 0$$

By Cramer's Rule, (1^{st} and 2^{nd})

$$\frac{x}{\begin{vmatrix} 1 & 2 \\ -6 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -5 & 2 \\ 0 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -5 & 1 \\ 0 & -6 \end{vmatrix}}$$

$$\frac{x}{13} = \frac{-y}{-5} = \frac{z}{30} = t$$

$x = 13t$

$y = 5t$

$z = 30t$

$$X_1 = \begin{bmatrix} 13t \\ 5t \\ 30t \end{bmatrix} = t \begin{bmatrix} 13 \\ 5 \\ 30 \end{bmatrix} \quad \text{——— (1)}$$

For $\lambda = 0.4$,

$$A = \begin{bmatrix} -1-0.4 & 1 & 2 \\ 0 & -2-0.4 & 1 \\ 2 & 0 & -3-0.4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1.4 & 1 & 2 \\ 0 & -2.4 & 1 \\ 2 & 0 & -3.4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-1.4x + y + 2z = 0$$
$$0x - 2.4y + z = 0$$
$$2x + 0y - 3.4z = 0$$

By Cramer's Rule (2^{nd} and 3^{rd}),

$$\frac{x}{\begin{vmatrix} -2.4 & 1 \\ 0 & -3.4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 0 & 1 \\ 2 & -3.4 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 0 & -2.4 \\ 2 & 0 \end{vmatrix}} = t$$

$$\frac{x}{8.16} = \frac{-y}{-2} = \frac{z}{4.8} = t$$

$$x = 8.16t \qquad X_2 = \begin{bmatrix} 8.16t \\ 2t \\ 4.8t \end{bmatrix} = 2t \begin{bmatrix} 4.08 \\ 1 \\ 2.4 \end{bmatrix}$$
$$y = 2t$$
$$z = 4.8t$$

$$X_2 = \begin{bmatrix} 4.08 \\ 1 \\ 2.4 \end{bmatrix} \quad \text{— (2)}$$

For $\lambda = -2.4$,

$$\begin{bmatrix} -1-2.4 & 1 & 2 \\ 0 & -2-2.4 & 1 \\ 2 & 0 & -2.4-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3.4 & 1 & 2 \\ 0 & -4.4 & 1 \\ 2 & 0 & -5.4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3.4x + y + 2z = 0$

$0x - 4.4y + z = 0$

$2x + 0y - 5.4z = 0$

By Cramer's Rule (1st and 2nd)

$$\frac{x}{\begin{vmatrix} 1 & 2 \\ -4.4 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -3.4 & 2 \\ 0 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -3.4 & 1 \\ 0 & -4.4 \end{vmatrix}} = t$$

$$\frac{x}{9.8} = \frac{-y}{-3.4} = \frac{z}{14.96} = t$$

$x = 9.8t$

$y = 3.4t$

$z = 14.96t$

$$X_3 = \begin{bmatrix} 9.8t \\ 3.4t \\ 14.96t \end{bmatrix} = t \begin{bmatrix} 9.8 \\ 3.4 \\ 14.96 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 9.8 \\ 3.4 \\ 14.96 \end{bmatrix} \quad \text{—— (3)}$$

From (1), (2), and (3)

$$X_1 = \begin{bmatrix} 13 \\ 5 \\ 30 \end{bmatrix} \quad X_2 = \begin{bmatrix} 4.08 \\ 1 \\ 2.4 \end{bmatrix} \quad X_3 = \begin{bmatrix} 9.8 \\ 3.4 \\ 14.96 \end{bmatrix}$$

Type 2: Matrix is symmetric, eigen values are non-repeated.

Q. find eigen value and eigen vector of following ~~order~~ matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Date

Ans. The characteristic eq^n of A is:

$|A - \lambda I| X = 0$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} = 0$$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

$S_1 = 8 + 7 + 3 = 18$

$S_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$

$= 5 + 20 + 20 = 45$

$|A| = 8(21 - 16) + 6(-18 + 8) + 2(24 - 14)$

$= 8 \times 5 + 6 \times -10 + 2 \times 10$

$= 40 - 60 + 20$

$= 0$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\lambda^2 - 18\lambda + 45 = 0$

$\lambda^2 - 15\lambda - 3\lambda + 45 = 0$

$\lambda(\lambda - 15) - 3(\lambda - 15) = 0$

$(\lambda - 15)(\lambda - 3) = 0$

$\lambda = 15, 3, 0$

$\therefore$ Eigen values are 0, 3, 15.

Page No.
Date

For $\lambda = 0$,

$$\begin{bmatrix} 8-0 & -6 & 2 \\ -6 & 7-0 & -4 \\ 2 & -4 & 3-0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ -2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x - 6y + 2z = 0$$
$$-6x + 7y - 4z = 0$$
$$-2x - 4y + 3z = 0$$

By Cramer's Rule (1^{st} and 2^{nd}),

$$\frac{x}{\begin{vmatrix} -6 & 2 \\ 7 & -4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 8 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}} = t$$

$$\frac{x}{10} = \frac{-y}{-20} = \frac{z}{20} = t$$

$x = 10t$
$y = 20t$
$z = 20t$

$$X_1 = \begin{bmatrix} 10t \\ 20t \\ 20t \end{bmatrix} = 10t \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

For $\lambda = 3$,

$$\begin{bmatrix} 8-3 & -6 & 2 \\ -6 & 7-3 & -4 \\ 2 & -4 & 3-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x - 6y + 2z = 0$$
$$-6x + 4y - 4z = 0$$
$$2x - 4y + 0z = 0$$

~~By Crammer's Rule (2^{nd} and 3^{rd})~~

$$\cancel{\frac{x}{\begin{vmatrix} 4 & -4 \\ -4 & 0 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -6 & -4 \\ 2 & 0 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -6 & 4 \\ 2 & -4 \end{vmatrix}} = t}$$

$$\cancel{\frac{x}{-16} = \frac{-y}{8} = \frac{z}{16} = t}$$

$\cancel{x = -16t}$

~~By Crawmer's Rule (~~

Q- $x + 4y - 6z = 1$
$2x - 3y + 5z = 1$
$3x + y - z = 2$

$$\begin{bmatrix} 1 & 4 & -6 \\ 2 & -3 & 5 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 2 & -3 & 5 & 1 \\ 3 & 1 & -1 & 2 \end{array}\right]$$

Apply $R_2 - 2R_1$ and $R_3 - 3R_1$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & -11 & +17 & -1 \end{array}\right]$$

Apply $R_3 - R_2$,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 2 <$ no. of variable.
$\therefore$ There are infinite solns.

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

eigen value
eigen vector smallest eig

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

$S_1 = 8$

$S_2 = (3-8) + (4-4) + (12+10) = -5+22 = 17$

$|A| = 4(3-8) - 2(-5+4) - 2(-20+6)$

$= 4 \times (-5) - 2(-1) - 2(-14)$

$= -20 + 2 + 28$

$= 10$

$\lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$

$-1 - 8 - 17 - 10$

$1 - 8 + 17 - 10 = 18 - 18 = 0$ 1, 2, 5

$(\lambda - 1)(\cancel{\lambda^2}$

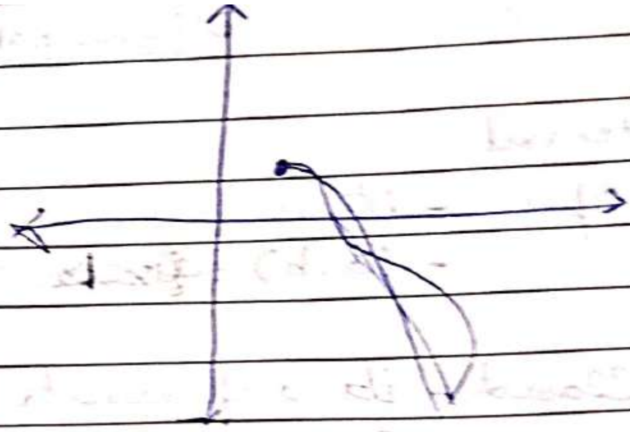
1	1	-8	17	-10
	1	-1		
	1	-9		

$\lambda = 1$

$1 - 8 + 17 - 10 = 0$

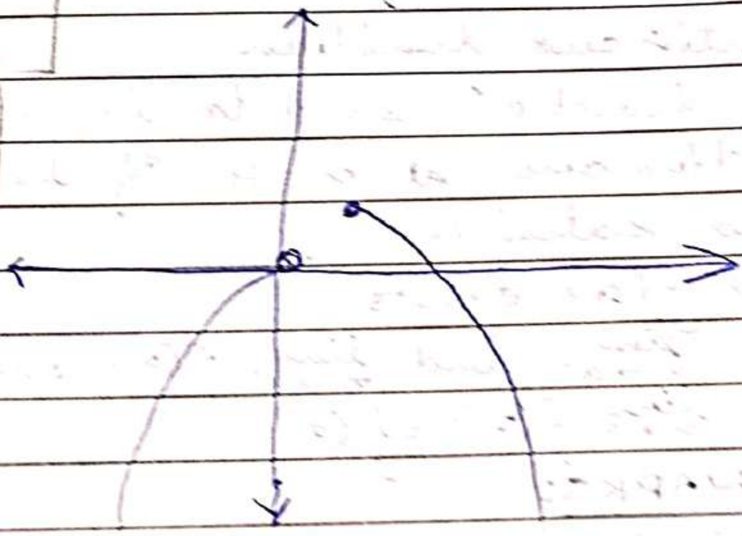
$$A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 2-1 & -2 \\ -2 & 5-1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$



$$A = \begin{bmatrix} 2-6 & -2 \\ -2 & 5-6 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}$$



$$\lim_{x \to 0} \frac{1}{x}$$